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On the Minimum Number of Units in Heat Exchanger Networks

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ABSTRACT: The minimum number of units in a network has been proposed to be obtained using a very simple formula using graph theory (Hohmann, E. C. Ph.D. Thesis, University of Southern California, Los Angeles, CA, 1971.; Linnhoff, B. et al. *Comput. Chem. Eng.* **1979**, *3*, 295). This is done, usually, assuming that thermodynamic feasibility holds, especially in Pinch technology, where it is applied above and below the pinch, but also for cases where the pinch is ignored. While the failure of this formula is informally known in the community, to our knowledge no realistic counterexample was presented before. We provide such an industrial counterexample. In addition, we propose to use a recently developed MILP model (Barbaro, A., and Bagajewicz, M. *Comput. Chem. Eng.* **2005**, *29*, 1945) that guarantees finding the global minimum number of exchangers. Finally, we point out the large number of alternative solutions that real industrial problems may exhibit.

1. INTRODUCTION

This article addresses the well-known formula for the minimum number of units that was proposed by Linnhoff et al.[1] and that has been used for the targeting of grassroots and retrofit designs of heat exchanger networks by enthusiasts of Pinch technology. We are not enthusiasts of the use of Pinch technology in all cases, but we recognize that fairly good results can be obtained for certain small systems, but we also know that it does not work well for other larger systems, especially crude units where there are significant problems.[2] However, because there are still enthusiasts of this procedure and also because it works actually well for small systems, we present a counterexample where the minimum number of exchangers rule is violated. This has some impact in targeting procedures that needs to be accounted for.

The original paper by Linnhoff et al.[1] argues that one can use results from graph theory to predict the minimum number of units in a heat exchanger network. The formula derived is

$$N = N_S + L - S \tag{1}$$

where N is the number of units, N_S is the total number of hot and cold streams as well as utilities, L is the number of "independent loops", and S is the number of independent subsystems that can be identified. They illustrate this concept by aligning hot streams and hot utilities in one row and cold streams and cold utilities in another and establishing connections among them. We reproduce their three examples in Figure 1 for $N_S = 6$.

Quite clearly, one achieves the minimum number of units by setting $L = 0$ and by identifying the maximum number of independent subsystems. This can be accomplished automatically by testing all partial sums of heat available from hot streams against all partial sums of heat demand from the cold streams. If any of these tests results in equal total values, then the system has at least two subsystems. One can continue with the same test for each subsystem.

Systems with two independent subsystems or more are rare in practice anyway. There is, however, an additional constraint: In order to apply the above formula, every connection between hot and cold streams has to be thermodynamically feasible. Linnhoff et al.[1] actually present an example where the above rule does not hold (Figure 2).

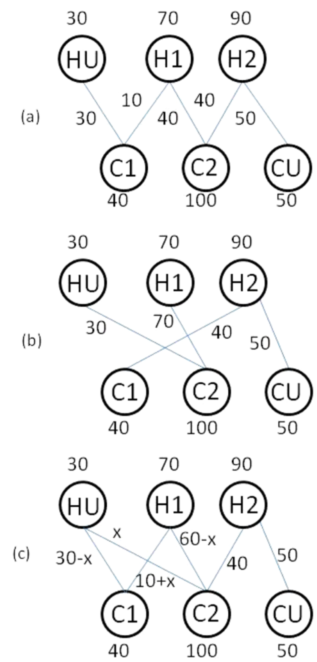

Figure 1. Number of units example.[1] (a) $L = 0$, $S = 1$, $N = 5$; (b) $L = 0$, $S = 2$, $N = 4$; (c) $L = 1$, $S = 1$, $N = 6$.

This example is actually incorrect and the minimum number of units ($N = 2$) can, after all, be achieved (Figure 3).

To deal with the thermodynamic feasibility, aside from assuming correctly that $L = 0$ should be used so that it targets

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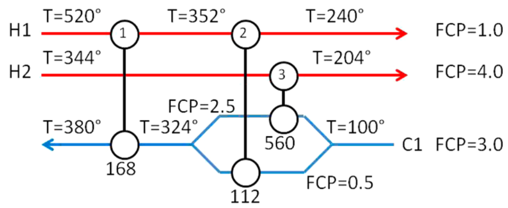

Figure 2. Example of rule violation based on Linnhoff et al.[1] example. Process−process matches exhibit a line connecting streams.

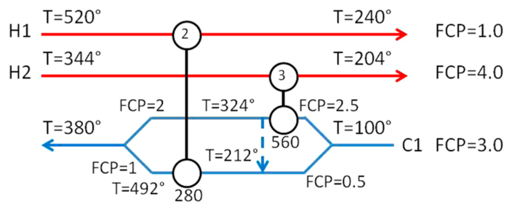

Figure 3. Solution with two units for the example presented by Linnhoff et al.[1] Process−process matches exhibit a line connecting streams.

the minimum number of units, pinch technology uses the formula above and below the pinch with the assumption that thermodynamic feasibility holds. Moreover, several books coined the expression

$$N = N_S - 1 \tag{2}$$

to be used above and below the pinch, which assumes no independent subsystems. Wood et al.[3] analyzed the validity of this formula for the whole system without dividing it in two subnetworks above and below the pinch. They concluded that in certain cases a flowsheet can be built in which the predicted number of units using the whole system information can be achieved, using bypasses that are similar to those shown in Figure 3. The work did not address fully the effect of thermodynamic constraints the way we present them here.

Aside from the aforementioned analysis by Linnhoff et al.,[1] this formula is cited by Kemp[4] with some discussion as to what is the result when applied ignoring the pinch, implying the breaking of loops, called "network relaxation." Kemp[4] never points out the work of Wood et al.[3] or the existence of the thermodynamic constraint. Next, Knopf[5] cites the original proponent[6] with no further discussion. Shenoy[7] cites it too without explanation but citing Linnhoff et al.[1]

In addition, several undergraduate level books on design present the formula: Towler and Sinnott[8] as well as Smith[9] state it without any clarifications also citing Hohmann.[6] Douglas[10] cites the rule by generalizing from examples but discusses thermodynamic feasibility citing the "Effect of the Pinch" and discussing it in the context of the second law. There is no discussion of thermodynamic feasibility on each side of the pinch. Interestingly, Biegler et al.,[11] state that the formula "$N - 1$" (they do not mention loops or subsystems) "is only an estimate". And they add: "we can sometimes do better and sometimes have to do worse", without giving a reason. Seider et al.[12] also cites the formula as proposed by Hohmann[6] but elaborates no further regarding feasibility. On the other hand, Turton et al.[13] adopt the very conservative advice to connect hot and cold streams with the minimum number of lines, implicitly assuming all connections are thermodynamically feasible. In all of the aforementioned books and textbooks, the implicit assumption is that thermodynamic feasibility holds if the formula is applied above and below the pinch separately.

Finally, it is worth noting that sometimes the formula "$N - 1$" gives an upper bound of the number of units when there is more than one subsystem ($S > 1$). This, for example, happens when two streams have exactly the same heat duty.

The article is organized as follows: We first discuss algorithms for upper bound prediction, and then we provide one method that predicts the minimum exactly.

2. BOUNDS

Cerda and Westerberg[14] presented a transporation-based model that can provide the minimum number of matches. In the same year, Papoulias and Grossmann[15] presented a model that also determines the minimum number of matches. These algorithms, however, do not predict the minimum number of exchangers. They only give a lower bound on the number of exchangers. Furman and Sahinidis[16] considered the idea of placing an upper bound on the number of matches in their exploration of approximation algorithms for the minimum number of matches problem.

3. METHOD FOR EXACT PREDICTION

Barbaro and Bagajewicz[17] presented an MILP model to design heat exchanger networks. The model equations are based on a transportation/trans-shipment model (Figure 4).

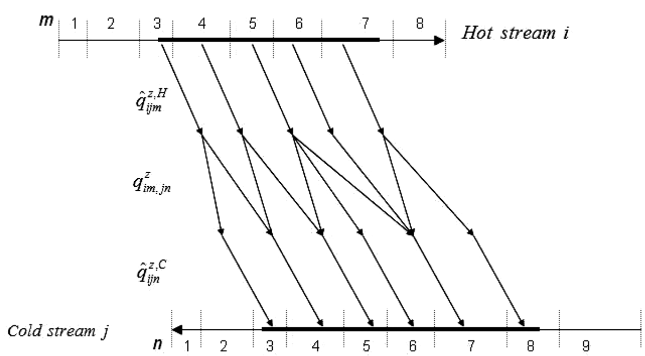

Figure 4. Basic scheme of the MILP model[17] indicating intervals m on hot stream i matching with intervals n on cold stream j.

In this model, aside from the energy balance equations, there are equations that can identify consecutive intervals in both hot and cold streams that are connected and thus determine that only one exchanger is needed.

This unit identification and counting requires the introduction of new variables. Consider hot stream i, a binary variable Y_{ijm}^{H}, and two continuous positive variables K_{ijm}^{H} and $\hat{K}_{ijm}^{\mathrm{H}}$. The binary variable Y_{ijm}^{H} indicates that there is a match between stream i at interval m sending heat to some intervals of stream j. In turn, K_{ijm}^{H} and $\hat{K}_{ijm}^{\mathrm{H}}$ indicate the beginning and end of a string of intervals for which the binary variable Y_{ijm}^{H} is active, that is, $Y_{ijm}^{\mathrm{H}} = 1$.

The equations for determining K_{ijm}^{H}, the beginning of a unit, are as follows.

$$K_{ijm}^{\mathrm{H}} \geq Y_{ijm}^{\mathrm{H}} \qquad m = 1 \tag{3}$$

$$K_{ijm}^{\mathrm{H}} \leq 2 - Y_{ijm}^{\mathrm{H}} - Y_{ijm-1}^{\mathrm{H}} \qquad m \neq 1 \tag{4}$$

$$K_{ijm}^{\mathrm{H}} \leq Y_{ijm}^{\mathrm{H}} \qquad m \neq 1 \tag{5}$$

$$K_{ijm}^{\mathrm{H}} \leq Y_{ijm}^{\mathrm{H}} - Y_{ijm-1}^{\mathrm{H}} \qquad m \neq 1 \tag{6}$$

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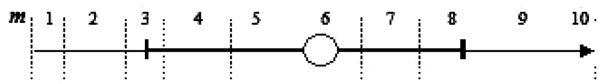

m	$Y^{z,H}_{ijm}$	$K^{z,H}_{ijm}$	$\hat{K}^{z,H}_{ijm}$
1	0	0	0
2	0	0	0
3	1	1	0
4	1	0	0
5	1	0	0
6	1	0	0
7	1	0	0
8	1	0	1
9	0	0	0

Figure 5. Example of heat exchanger definition over intervals m of hot stream. Indicates the unit with matches, Y^{H}_{ijm}, its beginning, K^{H}_{ijm}, and its end, $\hat{K}^{H}_{ijm}$.

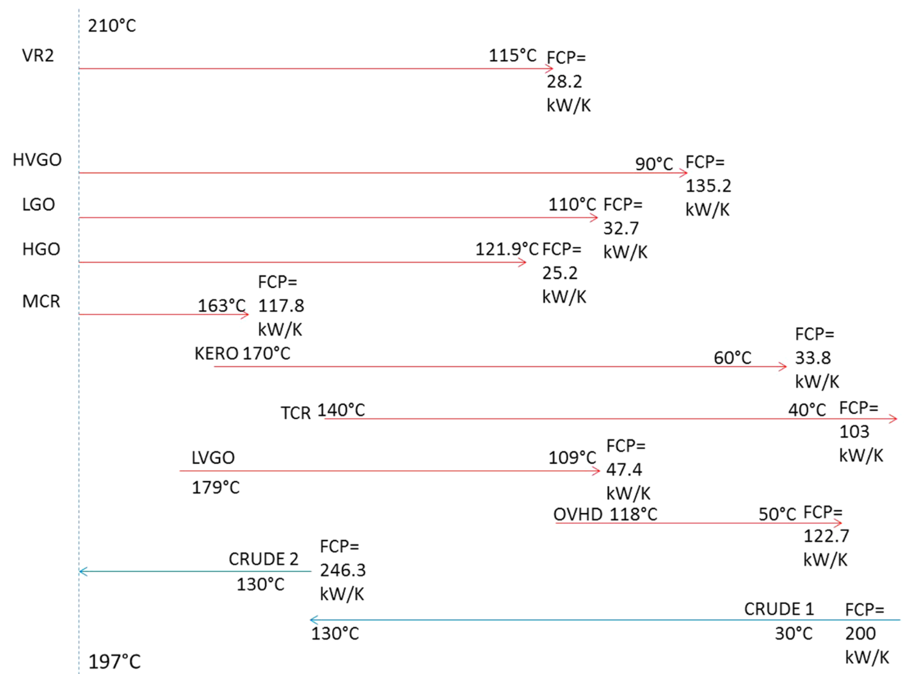

Figure 6. System below the pinch for a crude unit. Below the pinch, 11 process streams and 1 utility exist. The minimum number of units according to eq 2 is 11.

For the determination of the value of $\hat{K}^{H}_{ijm}$, the end of the exchanger, we use

$$\hat{K}^{H}_{ijm} \geq Y^{H}_{ijm} \qquad m = M \text{ (last interval)} \tag{7}$$

$$\hat{K}^{H}_{ijm} \leq 2 - Y^{H}_{ijm} - Y^{H}_{ijm+1} \qquad m \neq M \tag{8}$$

$$\hat{K}^{H}_{ijm} \leq Y^{H}_{ijm} \qquad m \neq M \tag{9}$$

$$\hat{K}^{H}_{ijm} \leq Y^{H}_{ijm} - Y^{H}_{ijm+1} \qquad m \neq M \tag{10}$$

We now illustrate how these constraints work. Consider the hot stream illustrated in Figure 5. The hot side of a heat exchanger involving stream i and stream j spans intervals 3 through 8. The values of Y^{H}_{ijm}, K^{H}_{ijm}, and $\hat{K}^{H}_{ijm}$ for this example are given in the table to the right of Figure 5. These numbers are consistent with the above set of constraints and are uniquely defined by them.

The model also considers exchangers in parallel, for which it has constraints that guarantee that the flow rate is the same in each branch. It also allows nonisothermal mixing and computes the areas as well as the utility loads. Thus, the model is complete, because it allows every split and considers nonisothermal mixing.

All the exchangers involving stream i can be obtained by adding all the K^{H}_{ijm} values, which are either 1 or zero. Thus, the total number of exchangers is given by

$$N = \sum_{i,j,m} \hat{K}^{H}_{ijm} \tag{11}$$

Minimizing N solves the problem globally and gives the absolute minimum number of exchangers.

The model being a transportation model, the number of units will in principle depend on the minimum temperature approach

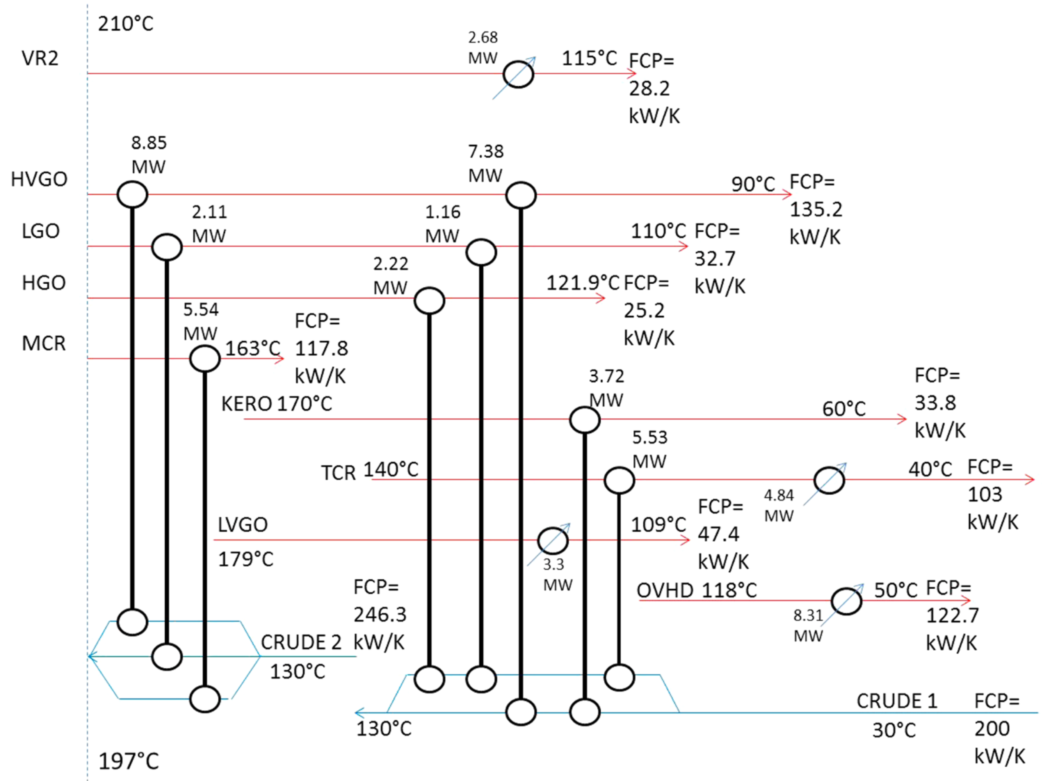

Figure 7. Counterexample: System below the pinch depicting required units (total of 12). Process-process matches exhibit a line connecting streams; utilities are single exchangers.

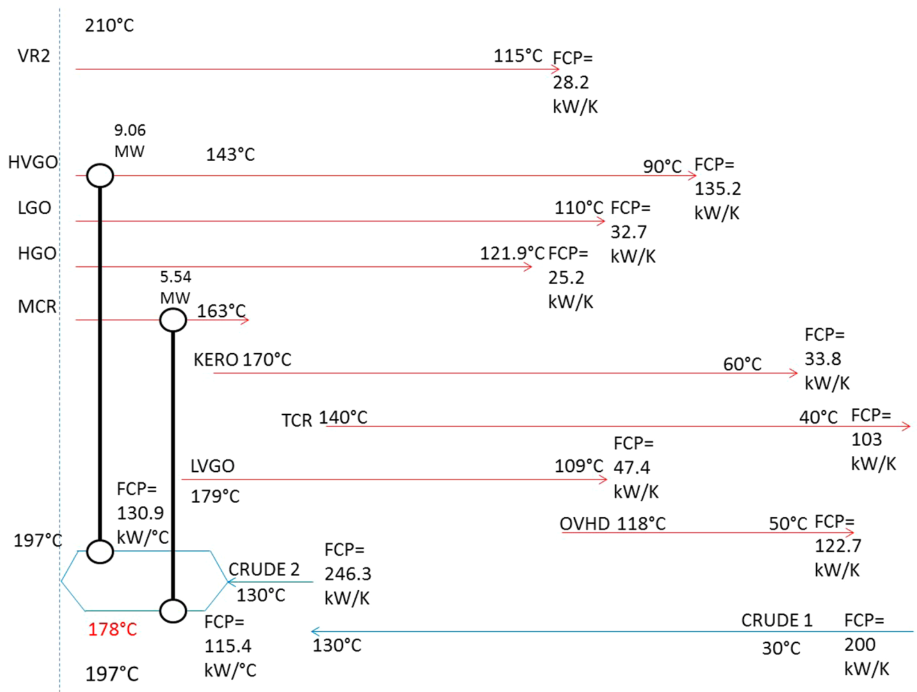

Figure 8. Counterexample: Explanation for requiring an additional exchanger. Process–process matches exhibit a line connecting streams.

($\Delta T_{\min}$) or heat recovery minimum approach temperature (HRAT). First, the model is linear and therefore cannot fall into any local minimum, meaning it gives the globally optimal answer. In addition, the model counts exchangers correctly. The proof is as follows: first, when there are several consecutive intervals exchanging heat between a hot stream and a cold stream, it is counted as one exchanger (as shown in Figure 5). If there is one interruption, that is, there are two sets of several consecutive intervals in the exchange, separated by an interval, or portion thereof, where the exchange is with some other cold stream, then the set of constraints will count two exchangers. Thus, the set of constraints counts correctly all the exchangers. Moreover, if the number of exchangers between these two streams is one and not two at the optimum, there is nothing preventing the rearrangement of heat transportation patterns to render such solution. Finally, the model allows for splits, nonisothermal mixing, and bypasses within any split, which covers all possibilities and therefore all networks are being considered.

4. EFFECT OF THERMODYNAMIC CONSTRAINTS

Furman and Sahinidis[16] provided a simple two hot, two cold streams, yet notional, counterexample to the $N-S$ rule, indicating that the actual minimum can be larger, therefore forcing cycles.

We now present an example for a much larger and closer to an industrial situation (crude unit) where the rule is also violated because of thermodynamic constraints. Consider a system below the pinch, which corresponds to a crude unit (Figure 6). There is no solution featuring 11 exchangers, which is, however, what formula 2 suggests. One solution is shown in Figure 7.

To understand the reason that the minimum number of exchangers is not achieved, the pinch design method must be revisited. First, at the pinch there are five hot streams reaching the pinch temperature (210 °C): HVGO, HGO, MCR, LGO, and VR2, which are candidates for pinch matches with one cold stream reaching the pinch temperature at 197 °C. This cold stream has a variable FCP, whose average between its initial temperature and the pinch is approximately 246.3 kW/K.

If the FCP rule is applied, that is, FCP (cold) ≥ sum of FCP (hot) for pinch matches, only two matches should be required: HVGO−Crude 2 and MCR−Crude 2, with the crude splitting in two branches and merging at the pinch. The combined average FCP of HVGO and MCR is 253 kW/K, which should be more than sufficient. No other combination of two hot streams has enough combined FCP to become a candidate and using more streams incurs already more matches than the minimum. However, if the $\Delta T_{\min}$ value chosen is low (we chose a value of 13 °C in this case) then three exchangers may be required to heat the Crude to 197 °C. MCR is the culprit, with a large FCP and small temperature range.

Figure 8 depicts the two pinch matches suggested by pinch technology. First, HVGO does not reach its target temperature of 90 °C. The FCP of the crude matched with HVGO cannot be increased because it would cause a $\Delta T_{\min}$ violation.

The HVGO outlet temperature is 13 °C ($\Delta T_{\min}$) larger than the Crude 2 starting temperature of 130 °C. Thus, the amount of heat available from HVGO is limited to 9060 kW. The rest (7442 kW) should be provided by MCR. However, because of its final temperature, MCR only has 5540 kW to give. This means that there is 1902 kW missing, which has to be provied by a third stream, making the "tick-off" rule impossible to be obeyed. Thus, using only two matches is not thermodynamically feasible, and this explains the extra exchanger.

We note here that the example above shows that the "$N-1$" rule is violated because of thermodynamic constraints. However, in this case, the model by Papoulias and Grossman[15] correctly identifies the minimum number of exchangers. Even though such a model is correct for this example, it is possible for it to fail by identifying matches between two streams that actually correspond to more than one unit.

5. MULTIPLE SOLUTIONS

This system of Figure 6 has 198 different solutions featuring 12 exchangers. We obtained these by using our MILP model excluding each solution found at a time, adding the following constraint each time a solution (s) is found.

$$\sum_{i,j,m \in B_s} \hat{K}^{\mathrm{H}}_{ijm} - \sum_{i,j,m \notin B_s} \hat{K}^{\mathrm{H}}_{ijm} \leq \mathrm{Card}(B_s) - 1 \tag{12}$$

where B_s is the set of variables $\hat{K}^{\mathrm{H}}_{ijm}$ that are equal to one in the current solution s. We also run the trans-shipment model proposed by Papoulias and Grossmann,[15] excluding solutions using a similar exclusion constraint. This trans-shipment model identified 211 solutions, 13 more than our model. These 13 extra solutions correspond to cases where the matches identified (12) do not correspond to the same number of exchangers, but rather a higher number, which was determined by our model. This large number of solutions renders inspection and rules-based methods like the pinch design method rather cumbersome to use, unless they are automated.

6. CONCLUSION

In this article, we show that the well-known and very popular formula to predict the minimum number of units in a heat exchanger network can fail because of thermodynamic constraints, even if the network is divided above and below the pinch. Although this failure was suspected by many and even pointed out by Furman and Sahinidis,[16] it is seldom mentioned in textbooks, creating the impression in students that the rule does not fail. It is imperative that clarifications be added. The trans-shipment model,[15] known to give the minimum number of matches, was tested, and it rendered also the same number of matches as those identified above for our example. We also point out that networks featuring the minimum number of exchangers can be identified with the MILP model we present above and can exhibit very large degeneracy rendering some step-by-step ruled based methods like the pinch design method, rather impractical, unless automated. Incidentally, the trans-shipment model identifies more matches than our MILP model. Those correspond to solutions where the number of exchangers is larger than the number of matches.

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Notes

The authors declare no competing financial interest.

■ REFERENCES

(1) Linnhoff, B.; Mason, D. R.; Wardle, I. Understanding Heat Exchanger Networks. *Comput. Chem. Eng.* **1979**, 3, 295−302.

(2) Bagajewicz, M.; Valtinson, G.; Nguyen, D. Retrofit of Crude Units Pre-Heating Trains: Mathematical Programming vs. Pinch Technology. *Ind. Eng. Chem. Res.* **2013**, *52* (42), 14913−14926.

(3) Wood, R. M.; Wilcox, R. J.; Grossmann, I. E. A Note on the Minimum Number of Units for Heat Exchanger Network Synthesis. *Chem. Eng. Commun.* **1985**, *39* (1−6), 371−380.

(4) Kemp, I. C. Data extraction and energy targeting. *Pinch Analysis and Process Integration*, 2nd ed.; Butterworth-Heinemann: Boston, MA, 2007; Chapter 3, pp 69−73.

(5) Knopf, F. C. Process Energy Integration. *Modeling, Analysis and Optimization of Process and Energy Systems*, 2nd ed.; John Wiley & Sons, Inc.: Hoboken, NJ, 2011; Chapter 13, p 328.

(6) Hohmann E. C. Optimum Networks for Heat Exchange. Ph.D. Thesis, University of Southern California, Los Angeles, CA, 1971.

(7) Shenoy, U. V. From targeting to supertargeting. *Heat Exchanger Network Synthesis: Process Optimization by Energy and Resource Analysis*; Gulf Publishing Company: Houston, 1995; Chapter 1, p 32.

(8) Towler, G.; Sinnott, R. K. *Chemical Engineering Design*; Butterworth-Heinemann: Amsterdam, Netherlands, 2008; Chapter 3, p 137.

(9) Smith, R. Heat Exchanger Networks III – Capital and Total Cost Targets. *Chemical Process Design and Integration*; John Wiley & Sons, Inc.: Hoboken, NJ, 2005; Chapter 17, p 387.

(10) Douglas, J. M. *Conceptual Design of Chemical Processes*; McGraw Hill: New York, 1988; Chapter 8, p 233.

(11) Biegler, L. T.; Grossmann, I. E.; Westerberg, A. W. *Systematic Methods of Chemical Process Design*; Prentice Hall: Upper Saddle River, NJ, 1997; Chapter 10, p 349.

(12) Seider, W. D.; Seader, J. D.; Lewin, D. R, Heat and Power Integration. *Product and Process Design Principles: Synthesis, Analysis and Evalutation*, 2nd ed.; John Wiley & Sons, Inc.: New York, 2004; Chapter 10, p 326.

(13) Turton, R.; Bailie, R.; Whiting, W.; Shaeiwitz, J. *Analysis, Synthesis, and Design of Chemical Processes*; Prentice Hall: Upper Saddle River, NJ, 1995; Chapter 19, p 535.

(14) Cerda, J.; Westerberg, A. W. Synthesizing Heat Exchanger Networks having Restricted Stream/Stream Matches Using Transportation Problem Formulations. *Chem. Eng. Sci.* **1983**, *38* (10), 1723–1740.

(15) Papoulias, S. A.; Grossmann, I. E. A Structural Optimization Approach in Process Synthesis- II Heat Recovery Systems. *Comput. Chem. Eng.* **1983**, *7* (6), 707–721.

(16) Furman, K. C.; Sahinidis, N. V. Approximation Algorithms for the Minimum Number of Matches Problem in Heat Exchanger Network Synthesis. *Ind. Eng. Chem. Res.* **2004**, *43* (14), 3554–3565.

(17) Barbaro, A.; Bagajewicz, M. New rigorous one-step MILP formulation for heat exchanger network synthesis. *Comput. Chem. Eng.* **2005**, *29* (9), 1945–1976.